BMRT-1 2001 Version A

Model Documentation

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July 29, 2008

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Chapter 1

Model Overview

BMRT-1 (Basic Model of Regional Trade 1-Region) is a constant returns, perfectly competitive model of Canada. Unlike the related BMRT model, Canada is represented as one region.

The structure of intermediate and final use is derived from the 2001 L-level national input-output table supplied by the Input-Output Division of Statistics Canada. The L-level table gives dramatically more commodity/sector detail than are available at the provincial level. Canada as a whole is modelled as a small open economy engaging trade with the rest of the world (ROW).

1.1 Implementation Details

BMRT-1 is a part of the on-going *Canadian Regional Economic Analysis Project* (CREAP) to promote policy modelling research on the Canadian economy. The model is distributed through the project web site at http://creap.wlu.ca. It is made available as a basic vehicle for using the CREAP data. It is our intention that others will modify and refine the model for their own purposes

BMRT-1 is written in the modelling language of GAMS and MPSGE. Because at least some of the users will be acquainted with GTAP and/or GTAPinGAMS, we have chosen to retain GTAP notations wherever possible. The following background information provides a brief description of modelling tools related to BMRT-1:

- GAMS (General Algebraic Modeling System) http://www.gams.com: its modelling language and optimization software are used for research in economics, mathematics, science and engineering.

- MPSGE (Mathematical Programming System for General Equilibrium) http://www.mpsge.org: popular among economists for ease of use, fast and efficient algorithms, flexibility in model design, and ability to run as a subsytem inside GAMS.
- GTAP (Global Trade Analysis Project) http://www.gtap.org: a consortium of universities, governments, and research institutions to develop models and data for policy research in global trade.
- GTAPinGAMS http://www.mpsge.org/gtap6: Rutherford's rendition of GTAP into GAMS/MPSGE format.

1.2 Additional Tax Features

BMRT-1 accommodates a wide range of taxes including the following common tax categories:

- direct (income) taxes distinguished by factors,
- indirect taxes on final demands,
- indirect taxes on intermediate uses.

Chapter 2

Algebraic Representation of BMRT-1

This chapter provides an algebraic representation of basic components in BMRT-1, namely, consumers, producers, and trade.

2.1 Model Dimensions

The model has the following dimensions:

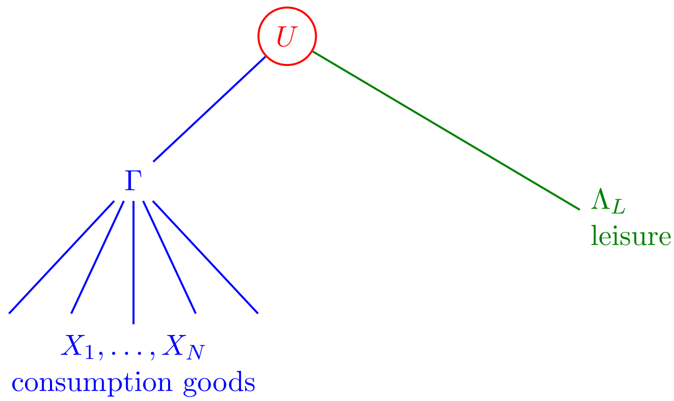
- K *productive sectors* indexed by $k = 1, \ldots, K$.
- N *produced goods/services* indexed by $i = 1, \ldots, N$.
- F *primary factors of production* indexed by $f = 1, \ldots, F$. In particular, labour/leisure is identified by $f = L$ and non-labour factors are identified by $f \neq L$.

2.2 Representative Agents and Final Demand

Preferences are represented as nested CES functions of leisure and Armington composites of all the produced goods (see figure 2.1). Private and public consumption as well as private and public investment are all lumped together in this treatment.

In MPSGE, imbalances in the balance of payment are handled by giving countries with a deficit an endowment of the numéraire consumption composite (see example m4_2s.gms in the Markusen tutorial available from the MPSGE web site). Because Canada has an balance of trade surplus, it is given a negative endowment (ξ) of 'foreign exchange' to help bring national expenditures into line with the national income.

Figure 2.1: Tree Diagram of Preference Structure



- Preference structure
  The utility function U is defined in terms of the composite good Γ and leisure Λ_L. The composite good Γ is in turn defined in terms of the consumer demands for final goods X_i ($i = 1, \ldots, N$).

$$U = U(\Gamma, \Lambda_L) \quad (2.1)$$
$$\Gamma = \Gamma(X_1, \ldots, X_N) \quad (2.2)$$

- Consumer income
  Consumer income Y comes from 3 sources, namely, factor endowment income (net of leisure), foreign exchange endowment Z (in domestic currency), and government tax transfer T. Factor prices are denoted by w_f and factor endowments (net of leisure) are denoted by $E_f - \Lambda_f$. Note that Λ_f are all zero except for labour/leisure (i.e., $\Lambda_f = 0, \forall f \neq L$).

$$Y = \sum_{f=1}^{F} w_f(E_f - \Lambda_f) + Z + T \quad (2.3)$$

- Budget constraint
  On the expenditure side (LHS) of the budget constraint, $\Pi_{i\mathcal{F}}$ are the consumer prices of final goods X_i which are different from the producer

prices Π_i by the ad-valorem taxes t_i (equation 2.5).

$$\sum_{i=1}^{N} \Pi_{i\mathcal{F}} X_i \leq Y \quad (2.4)$$

$$\Pi_{i\mathcal{F}} = \Pi_i(1 + t_i) \quad (2.5)$$

2.3 Firms and Production

The production side of the economics has a nesting structures with standard economic functional forms such as Leontief (fixed proportions), Cobb-Douglas, CES (constant elasticity of substitution), and CET (constant elasticity of transformation). Since the model is implemented in MPSGE, it is trivial to switch from one functional form to another. In most cases, switching these functional forms requires no more than a change in the elasticity value (e.g., zero for Leontief, unity for Cobb-Douglas, and any other value for CES/CET).

- Multi-output production
  Each sector k has a constant returns to scale multi-output production technology (equation 2.6) which takes inputs (composite intermediate goods $\tilde{A}_k$ and value-added Υ_k) on the RHS and produces several outputs ('unfinished' output goods y_{ik}) on the LHS (see figure 2.2).

$$Q_k(y_{1k} \dots y_{Nk}) = F_k(\tilde{A}_k, \Upsilon_k) \quad (2.6)$$

On the input side (RHS of equation 2.6), input substitution is further nested. The composite intermediate goods $\tilde{A}_k$ is broken into intermediate inputs $A_{1k}, \dots, A_{Nk}$ (equation 2.7) and the value-added Υ_k broken into primary factor inputs $b_{1k}, \dots, b_{Fk}$ (equation 2.8)

$$\tilde{A}_k = \tilde{A}_k(A_{1k}, \dots, A_{Nk}) \quad (2.7)$$

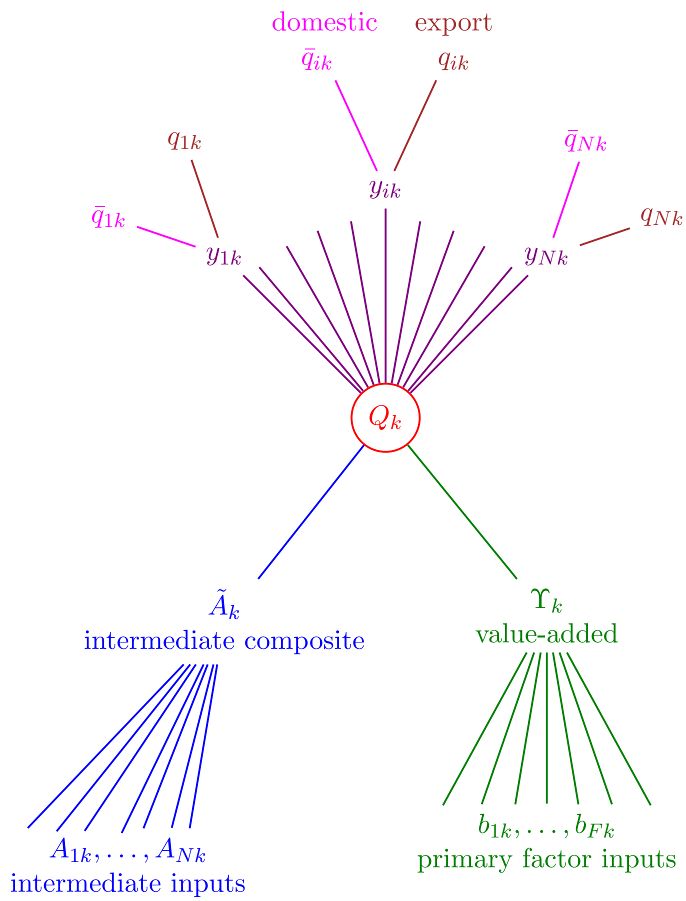
$$\Upsilon_k = \Upsilon_k(b_{1k}, \dots, b_{Fk}) \quad (2.8)$$

On the output side (LHS of equation 2.6), there is a CET transformation among the various 'unfinished' output goods y_{ik}. These unfinished goods are further transformed into finished goods for the *domestic* market $\bar{q}_{ik}$ and finished goods for the *export* market q_{ik} (see equation 2.9).

$$y_{ik} = y_{ik}(\bar{q}_{ik}, q_{ik}) \quad (2.9)$$

By default, the elasticity of transformation between domestic and export varieties of a given good is set to a finite positive value. If this is set

Figure 2.2: Tree Diagram of Production Structure



to infinity, the model produces only one variety of each distinct good. In this case, the same produced good is sold in the export or domestic market.

- Prices
  Equation 2.10 defines the producer factor price ω_f by adding ad-valorem factor tax λ_f to the market price w_f for factors. Similarly, equation 2.11 defines the producer intermediate price Π_{ik} by adding ad-valorem intermediate tax σ_k to the market price Π_i for intermediate inputs.

$$\omega_f = w_f(1 + \lambda_f) \quad (2.10)$$

$$\Pi_{ik} = \Pi_i(1 + \sigma_k) \quad (2.11)$$

- Zero profit conditions
  For each sector k, the total revenue from finished goods (domestic and export) (LHS of equation 2.12) must equal the total cost of intermediate inputs and primary factors (RHS of equation 2.12)

$$\sum_{i=1}^{N} p_i\,\bar{q}_{ik} + \sum_{i=1}^{N} \tilde{p}_i\,q_{ik} \quad = \quad \sum_{i=1}^{N} \Pi_{ik}\,A_{ik} + \sum_{f=1}^{F} \omega_f\,b_{fk} \qquad (2.12)$$

2.4 Trade and Import Aggregation

The finished goods discussed in equation 2.9 (namely, $\bar{q}_{ik}$ for domestic market or q_{ik} for export market) are the goods purchased to create the Armington aggregates to be discussed next. The intermediate and final Armington composites are the same. In other words, when final consumers buy electrical products, they get the same share of Canadian to imported electrical products as intermediate (industry) users do. This simplifies the model somewhat and we are unaware of any data to the contrary anyway.

- Domestic-import aggregation
  Equation 2.13 describes the domestic-import Armington aggregation. The nesting structure separates world imports M_i^w from domestic goods x_i (see figure 2.3).

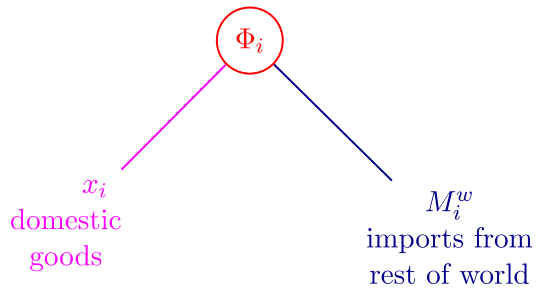
$$\Phi_i \quad = \quad \Phi_i(x_i, M_i^w) \qquad (2.13)$$

- Trading regions
  There are two trading regions in the model, namely, Canada (denoted by CAN or superscript c) and Rest of World (denoted by ROW or superscript w). Being a small open economy, Canada takes actions from ROW, e.g., prices, quantities, and taxes as exogenously given (denoted with a tilde $\tilde{}$ like $\tilde{p}_i^w \equiv \tilde{p}_i$).

  The double index sd provides a more precise notation by distinguishing a *source* s from a *destination* d. For example, τ_x^{cw} denotes the export tax imposed by CAN on goods originating from CAN to ROW and $\tilde{\tau}_m^{cw}$ denotes the import tax imposed by ROW on that same goods from CAN to ROW.

  When there is no danger of ambiguity, we use the simpler notations $\tau_x \equiv \tau_x^{cw}$ for Canadian export tax and $\tilde{\tau}_m \equiv \tilde{\tau}_m^{cw}$ for ROW import tax. Similarly, we use $\tau_m \equiv \tau_m^{wc}$ for Canadian import tax and $\tilde{\tau}_x \equiv \tilde{\tau}_x^{wc}$ for ROW export tax.

Figure 2.3: Tree Diagram of Domestic-Import Aggregation



- Trade taxes
  Equations 2.14–2.17 incorporate trade taxes into price relations for the two trading regions CAN and ROW. Negative values refer to subsidies such as export or import subsidies. There are no trade taxes on goods that stay within the same region (e.g., inside CAN or inside ROW). We have prices (including trade taxes) for export goods from CAN to ROW and import goods from ROW to CAN as follows:

$$\rho_i^{cw} = p_i^c(1+\tau_x^{cw})(1+\tilde{\tau}_m^{cw}) \quad (2.14)$$
$$\rho_i^{wc} = \tilde{p}_i^w(1+\tilde{\tau}_x^{wc})(1+\tau_m^{wc}) \quad (2.15)$$

  or in simpler notations

$$\rho_{x_i} = p_i(1+\tau_x)(1+\tilde{\tau}_m) \quad (2.16)$$
$$\rho_{m_i} = \tilde{p}_i(1+\tilde{\tau}_x)(1+\tau_m) \quad (2.17)$$

2.5 Rest of World

ROW is represented by two types of activities. Export activities (1 per exported commodity) take inputs of foreign exchange and produce ROW exports. Import activities (1 per imported commodity) take inputs of Canadian exports and produce foreign exchange. Equations 2.18 and 2.19 are the 'technologies' embodying the *small open economy* assumption in these two activities.

- Import technology
  Equation 2.18 describes the import technology where α_i is the constant

foreign exchange price of importing goods i from ROW (source w) to CAN (destination c). It could also be thought of as the quantity of foreign exchange required to 'produce' one unit of imports from ROW to CAN. Thus ν_i is the total foreign exchange required to import the amount $m_i^{wc} \equiv m_i$ from ROW to CAN.

$$\nu_i = \alpha_i m_i \tag{2.18}$$

- Export technology
  Similarly, equation 2.19 describes the export technology where γ_i the price (in foreign currency) of exporting goods i from CAN (source c) to ROW (destination w). Thus μ_i is the total foreign exchange earned from exporting the amount M_i^w from CAN to ROW.

$$\mu_i = \gamma_i M_i^w \tag{2.19}$$

- Foreign exchange rate
  Equation 2.20 links the price $\tilde{p}_i^w \equiv \tilde{p}_i$ of imports (in domestic currency) with the price α_i of imports (in foreign currency) through the foreign exchange rate Ω.

$$\tilde{p}_i = \alpha_i \Omega \tag{2.20}$$

- Zero profits
  Equation 2.21 describes the zero profit condition of the import technology where the cost and value of imports are equal in the same currency.

$$\Omega \nu_i = \tilde{p}_i m_i \tag{2.21}$$

2.6 Accounting Identities

The determination of national income in equation 2.3 requires a definition for the trade deficit/surplus (Z) and government tax transfer (T).

- Balance of payments
  The amount of foreign exchange endowment Z (in domestic currency) given to consumers is defined to be the domestic currency equivalence of the amount of foreign exchange ξ (in foreign currency) required to bring the national income into line with the national income.

$$Z = \Omega \xi \tag{2.22}$$

- Government tax revenue
  The amount of government tax transfer T given to consumers is defined to be the total tax revenues collected from various sources.

$$\begin{aligned} T \quad = \quad & \underbrace{\sum_{i=1}^{N} \Pi_i \, X_i \, t_i}_{\text{final demand taxes}} \quad + \underbrace{\sum_{k=1}^{K} \sum_{f=1}^{F} w_f \, b_{fk} \, \lambda_f}_{\text{factor input taxes}} \\ & + \underbrace{\sum_{k=1}^{K} \sum_{i=1}^{N} \Pi_i \, A_{ik} \, \sigma_k}_{\text{intermediate input taxes}} \\ & + \underbrace{\sum_{i=1}^{N} \tilde{p}_i (1 + \tilde{\tau}_x) \, m_i \, \tau_m}_{\text{import taxes}} \\ & + \underbrace{\sum_{i=1}^{N} p_i \, M_i^w \, \tau_x}_{\text{export taxes}} \end{aligned} \tag{2.23}$$

2.7 Market Clearing

Distinct markets exist for a number of composite commodities (those that are actually consumed by producers or consumers) as well as the produced goods from which they are derived. Factors of production are mobile between sectors domestically but internationally immobile. At equilibrium, the following market clearing conditions must be satisfied:

- Armington composites
  Equation 2.24 requires that the total demand for Armington composites must equal its total supply.

$$\sum_{k=1}^{K} A_{ik} + X_i \quad = \quad \Phi_i \tag{2.24}$$

- Finished goods (domestic)
  Equation 2.25 requires that the total demand for finished goods for domestic usages must equal its total supply from all sectors.

$$x_i \quad = \quad \sum_{i=1}^{K} \bar{q}_{ik} \tag{2.25}$$

- Finished goods (export)
  Equation 2.26 requires that the total demand of finished goods for export to ROW must equal its total supply from all sectors.

$$M_i^w = \sum_{i=1}^{K} q_{ik} \tag{2.26}$$

- Primary factors
  Equation 2.27 requires that the total demand of factors by all sectors must equal the total factor endowment (net of leisure).

$$\sum_{k=1}^{K} b_{fk} = E_f - \Lambda_f \tag{2.27}$$

- Foreign exchange
  Equation 2.28 requires that the total demand for foreign exchange (consumer endowment and imports of goods) must equal the total supply of foreign exchange from export earnings. In other words, the balance of payment must be in equilibrium.

$$\xi + \sum_{i=1}^{N} \mu_i = \sum_{i=1}^{N} \nu_i \tag{2.28}$$

Appendix A

Reference Tables

Table A.1: Model Dimensions

K	Number of productive sectors	$k = 1, \ldots, K$
N	Number of produced goods/services	$i = 1, \ldots, N$
F	Number of primary factors of production	$f = 1, \ldots, F$

Table A.2: Trading Regions

Canada	CAN	denoted by superscript c
Rest of World	ROW	denoted by superscript w

Table A.3: Transactions (Quantities)

E_f	Endowment of factor f by representative agent
ξ	Endowment of foreign exchange by representative agent
Λ_L	Demand for leisure by representative agent
Γ	Demand for composite goods by representative agent
X_i	Demand for final goods i by representative agent
y_{ik}	Output of unfinished goods i by sector k
q_{ik}	Output of finished goods i (for domestic) by sector k
$\bar{q}_{ik}$	Output of finished goods i (for export) by sector k
Υ_k	Input of primary factor composites by sector k
b_{fk}	Input of primary factor f by sector k
$\tilde{A}_k$	Input of intermediate composites by sector k
A_{ik}	Input of intermediate input i by sector k
Φ_i	Domestic-import Armington composite of goods i
x_i	Domestic component of Armington composite of goods i
M_i^w	ROW import component of Armington composite of goods i
$m_i^{wc} \equiv m_i$	Import goods i from ROW (source w) to CAN (destination c)
$m_i^{cw} \equiv M_i^w$	Export goods i from CAN (source c) to ROW (destination w)
α_i	Foreign exchange needed to import 1 unit from ROW to CAN
ν_i	Foreign exchange needed to import m_i from ROW to CAN
γ_i	Foreign exchange earned by export 1 unit from CAN to ROW
μ_i	Foreign exchange earned by export M_i^w from CAN to ROW

Table A.4: Prices

w_f	Price of factor f (seller price)
ω_f	Price of factor f (with factor tax λ_f)
p_i	Price of finished goods i (domestic)
$\tilde{p}_i$	Price of finished goods i (export)
Π_i	Price of Armington composite goods
Π_{ik}	Price of Armington composite goods (with sector tax σ_k)
$\Pi_{i\mathcal{F}}$	Price of Armington composite goods (with consumer tax t_i)
$\rho_{x_i} \equiv \rho_i^{cw}$	Price of export goods from CAN to ROW (with trade taxes)
$\rho_{m_i} \equiv \rho_i^{wc}$	Price of import goods from ROW to CAN (with trade taxes)
Ω	Price of 'foreign exchange'

Table A.5: Tax Rates

t_i	Consumer tax on final demands for goods i
λ_f	Producer tax on factor f used in production
σ_k	Producer tax on intermediate inputs (net of subsidies) in sector k
τ_m	Import tax (subsidy if negative) on imports from ROW to CAN
τ_x	Export tax (subsidy if negative) on exports from CAN to ROW

Table A.6: Incomes and Expenditures ($)

Y	Total income of representative agent
T	Government tax transfer of representative agent
Z	Foreign exchange endowments (in domestic currency) of representative agent ($Z < 0$ if there is a balance of trade surplus and $Z > 0$ if there is a balance of trade deficit)

Table A.7: Equation Identifiers

Description	Number
Section 2.2: Representative Agent and Final Demand	
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