BMRT 2001 Version 3.0

Model Documentation

Trien T. Nguyen
Department of Economics
University of Waterloo

Tracy Snoddon
Randall M. Wigle[1]
Department of Economics
Wilfrid Laurier University

Revised: September 5, 2008

[1]Correspondence author: Randall M. Wigle, Department of Economics, Wilfrid Laurier University, 75 University Avenue West, Waterloo, Ontario N2L 3C5, Canada. Email: rwigle@wlu.ca.

Contents

List of Tables

List of Figures

Chapter 1

Overview

BMRT (*Basic Model of Regional Trade*) is a constant returns, perfectly competitive model of Canada. It features interprovincial and international trade. The structure of intermediate and final use is derived from the Canadian Input-Output (IO) data supplied by the Input-Output Division of Statistics Canada (StatCan). As a result, most productive sectors produce several outputs. Canada as a whole is modelled as a small open economy trading with the rest of the world (ROW).

Update Notice

The base provincial data used by BMRT (CREAP Version 3) was updated in September, 2008. Data sets derived from this data have the version identifier 'b'[1] This revision to the data revealed a bug in BMRT related to it's handling of the 'Government Abroad' accounts. As a result, Version 3b data sets require the September 5, 2008 version of BMRT.

1.1 Implementation Details

BMRT is a part of the on-going project CREAP (Canadian Regional Economic Analysis Project) to promote policy modeling research on the Canadian economy. The current version of the model is BMRT 2001 Version 3.0 which is based on the benchmark equilibrium data set CREAP 2001 Version 3.0 (see also the two companion documentations [2] [3]). Both the model and data can be downloaded from the CREAP web site (http://creap.wlu.ca). The model is made available as a basic vehicle for using the CREAP data. It is

[1] As in CREAP-DATA-2001-tiny-V3b.gdx.

our intention that others will modify and refine the model for their own purposes.

BMRT is written in the modeling language of GAMS and MPSGE. Because at least some of the users of BMRT will be acquainted with GTAP and/or GTAPinGAMS, we have chosen to retain GTAP notations wherever possible. The following background information provides a brief description of modeling tools related to BMRT:

- GAMS (http://www.gams.com) stands for *General Algebraic Modeling System*. Its modeling language and optimization software are used for research in economics, mathematics, science and engineering.
- MPSGE (http://www.mpsge.org) stands for *Mathematical Programming System for General Equilibrium*. It is popular among economists for its ease of use, fast and efficient algorithms, flexibility in model design, and ability to run as a subsytem inside GAMS.
- GTAP (http://www.gtap.org) stands for *Global Trade Analysis Project*. It is a joint effort of universities, governments, international organizations, and research institutions to develop models and data for policy research in global trade. Its GTAP global data sets are used in many applications in agriculture, trade and environment.
- GTAPinGAMS (http://www.mpsge.org/gtap6) is Thomas F. Rutherford's rendition of GTAP model into GAMS/MPSGE format to allow GAMS users to run GTAP data sets inside their familiar GAMS system.

1.2 Additional Tax Features

The current BMRT Version 3.0 accommodates a wide range of taxes including the following common tax categories:

- direct (income) taxes distinguished by factors,
- indirect taxes on final demands,
- indirect taxes on intermediate uses.

Chapter 2

Algebraic Model Representation

This chapter documents the model structure of `BMRT` with an algebraic representation of its basic components, namely, consumers, producers, trade, and markets. Appendix A contains reference tables on notations, variables, and equations.

2.1 Notes on Subscripts

In this chapter, the term *subscripts* will be used as a generic term to refer to either subscripts or superscripts. The model has five types of subscripts as follows:

a. *Regions/Provinces*
   - provinces are indexed by $p = 1, \ldots, P$;
   - regions (provinces plus rest of world) are indexed by $r, p, s, d = 1, \ldots, R$;
   - the index r denotes a region which can be either a province or rest of world (denoted by W) while the index p is specifically reserved for a province;
   - the double index sd is used when it is necessary to distinguish a *source* s from a *destination* d (e.g., interprovincial trade, world trade); in particular, the double indexes rr, ss mean that source and destination refer to the same region (e.g., exports and imports inside the same province).

b. *Productive Sectors*
   - sectors are indexed by $j, k = 1, \ldots, K$.

c. *Goods*
   - produced goods are indexed by $i = 1, \ldots, N$;

- in some variants of the model, the number of produced goods N is further divided into two subgroups with M goods belong to the *energy goods* subgroup and $N - M$ belong to the *non-energy goods* subgroup;
- in those cases, we follow the convention to identify the *first* $N - M$ goods as *non-energy goods* and the *last* M goods as *energy goods*.
- for example, suppose there are $N = 5$ goods (say, `G1`, `G2`, `G3`, `G4`, `G5`) with $M = 2$ energy goods and $N - M = 3$ non-energy goods; the non-energy goods will be the first $N - M = 3$ goods (i.e., `G1`, `G2`, `G3`) and the energy goods will be the last $M = 2$ goods (i.e., `G4`, `G5`) starting from index $N - M + 1 = 4$.

d. *Primary Factors*
- primary factors are indexed by $f = 1, \ldots, F$;
- in particular, labor (including leisure) is identified by $f = L$ and the remaining non-labor factors are identified by $f \neq L$.

e. *Levels of Governments*
- levels of governments are indexed by $\ell = 1, \ldots, L$;
- note that the same symbol L in two different cases: labor or leisure ($f = L$) and the number of levels of governments ($\ell = L$); there is little chance of ambiguity as the context is quite clear in each case.

2.2 Representative Agents

Equations (2.1–2.8) summarize the structure of final demands by a representative agent in each province.

$$U^p = U^p(\Gamma^p, \Psi^p, \Lambda_L^p) \qquad \text{(utility)} \tag{2.1}$$

$$\Gamma^p = \Gamma^p(X_1^p, \ldots, X_N^p) \qquad \text{(goods composite)} \tag{2.2}$$

$$\Psi^p = \Psi^p(\psi_1^p, \ldots, \psi_N^p) \qquad \text{(investment composite)} \tag{2.3}$$

$$Y^p = \sum_{f=1}^{F} \pi_f^p (E_f^p - \Lambda_f^p) + Z^p + T^p \qquad \text{(income)} \tag{2.4}$$

$$Y^p \geq \sum_{i=1}^{N} \pi_{i_{\mathrm{C}}}^p X_i^p + \sum_{i=1}^{N} \pi_{i_{\mathrm{I}}}^p \psi_i^p \qquad \text{(budget constraint)} \tag{2.5}$$

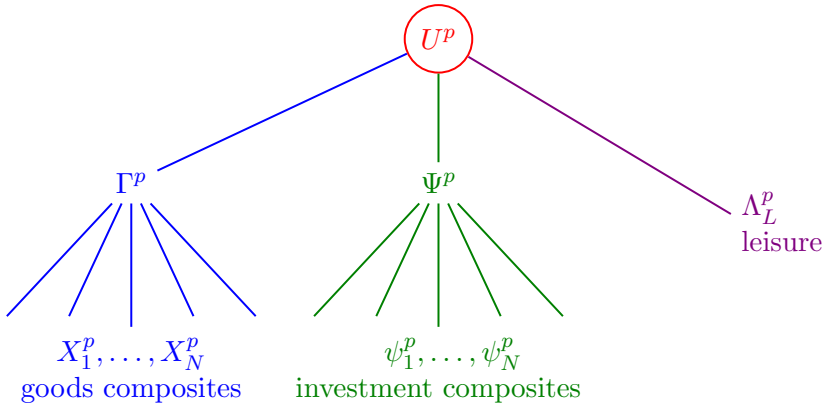
$$\pi_f^p = w_f^p (1 - td_f^p) \qquad \text{(factor price)} \tag{2.6}$$

$$\pi_{i_{\mathrm{C}}}^p = \pi_i^p (1 + te_{\mathrm{C}}^p) \qquad \text{(goods price)} \tag{2.7}$$

$$\pi_{i_{\mathrm{I}}}^p = \pi_i^p (1 + te_{\mathrm{I}}^p) \qquad \text{(investment price)} \tag{2.8}$$

Preferences are represented as nested CES functions of Armington goods composites, investment composites, and leisure (see figure 2.1). In situation involving labor like equation (2.4), the demand for leisure Λ_f^p is not applicable to factors other than labor (i.e., $\Lambda_f^p = 0$ for $f \neq L$). Private consumption and investment expenditures are distinguished from public expenditures by various levels of governments (see section 2.3).

Figure 2.1: Tree Diagram of Preference Structure



For each province, a representative agent purchases a vector of the Armington composite goods plus an investment composite. The investment composite is composed of a vector of purchases of the Armington composite goods. Equations (2.1, 2.2, 2.3) represent the nested preference structure while equations (2.4, 2.5) represent income determination and the budget constraint respectively. After-tax prices are defined in equations (2.6, 2.7, 2.8) for primary factors, goods consumption purchases, and investment purchases. Taxes included rates from all levels of governments.

Each province will have a trade imbalance that is accommodated either by other provinces or the ROW region.[1] To deal with this, each province with a balance of trade deficit is given a positive endowment of 'foreign exchange' (ξ^p) which allows them to pay for the extra imports. Provinces with a balance of trade surplus have a negative endowment of foreign exchange. This is deducted from their income available to spend.

[1] In `MPSGE`, imbalances in the balance of payments are handled by giving countries with a deficit an endowment of the numéraire consumption composite (see Example `M4_2S` in Markusen's Examples [1]).

Given the foreign exchange rate ε, the amount of foreign exchange endowment ξ^p is converted into the domestic currency by equation (2.9) and appears as Z^p in the budget constraint (2.5).

$$Z^p = \varepsilon\, \xi^p \tag{2.9}$$

Canada as a whole will have a trade imbalance. In other words, while some of the provincial deficits will be offset by other provincial surpluses there should be (in this case) an overall trade deficit. Thus, the sum of all the positive and negative provincial endowments of foreign exchange should be positive.

2.3 Governments

BMRT 2001 Version 3.0 provides more detailed coverage of the government sector than the previous version. The model identifies one provincial and one local government in each province, plus a federal government (indexed by $\ell = 1, ..., L$). Equations (2.10–2.12) summarize the structure of governments in the model.

$$\chi^{p\ell} = \chi^{p\ell}(\chi_i^{p\ell}, \ldots, \chi_N^{p\ell}) \qquad \text{(govt composite)} \tag{2.10}$$

$$\mathtt{GE}^{p\ell} = \sum_{i=1}^{N} \pi_{i_C}^{p} \chi_i^{p\ell} + \sum_{i=1}^{N} \pi_{i_I}^{p} \phi_i^{p\ell} + T^p \qquad \text{(govt expenditure)} \tag{2.11}$$

$$\mathtt{GE}^{p\ell} \leq \mathtt{GR}^{p\ell} \qquad \text{(govt constraint)} \tag{2.12}$$

Each level of government receives revenues identified as being attributable to them and purchases 'government' composites of goods. These government composites in equation (2.10) are cost-minimizing bundles of the goods, calibrated to the Government Net Current Expenditures from final demands. Equation (2.11) defines the expenditure side which consists of purchases of government composites ($\chi_i^{p\ell}$), government investments ($\phi_i^{p\ell}$), and revenue transfers (T^p) to consumers. The model thus allows governments to actively participate in the market for final goods and investments as much as the private consumers in section 2.2 above. Equation (2.12) represents the budget constraint in which government expenditures must not exceed government revenues received. Section 2.8 provides additional details on various components of government revenues.

In the benchmark, government expenditures often differ from government revenues. In that case, the difference is handled by a positive or

negative endowment of foreign exchange. The governments currently do not engage in intergovernmental transfers or transfers to/from persons. Increases or reductions in government revenues lead to corresponding changes in government expenditures. However, the model can be easily modified to achieve a fixed real government expenditure target.

While all levels of government receive indirect taxes, only the federal and provincial governments receive direct income taxes. The local and provincial governments spend their receipts on their province-specific government expenditures composite. The federal government buys fixed proportions by province of the province-specific government expenditures composites.

2.4 Firms and Production

The production side of BMRT 2001 Version 3.0 has not changed much from the previous version. Equations (2.13–2.21) summarize the production structure of the model.

$$\mathcal{F}_k^p(Y_{1k}^p,\ldots,Y_{Nk}^p) = F_k^p(A_{1k}^p,\ldots,A_{Nk}^p;\Upsilon_k^p) \quad \text{(multi-output)} \tag{2.13}$$

$$\Upsilon_k^p = \Upsilon_k^p(B_{1k}^p,\ldots,B_{Fk}^p) \quad \text{(value-added)} \tag{2.14}$$

$$Y_{ik}^p = Y_{ik}^p(Q_{ik}^p,\hat{Q}_{ik}^p) \quad \text{(domestic-export)} \tag{2.15}$$

$$p_f^p = w_f^p(1+tf_f^p) \quad \text{(factor price)} \tag{2.16}$$

$$p_{ik}^p = \pi_i^p(1+ti_{ik}^p) \quad \text{(input price)} \tag{2.17}$$

$$p_i^p = \pi_i^p(1-ty_i^p) \quad \text{(output price)} \tag{2.18}$$

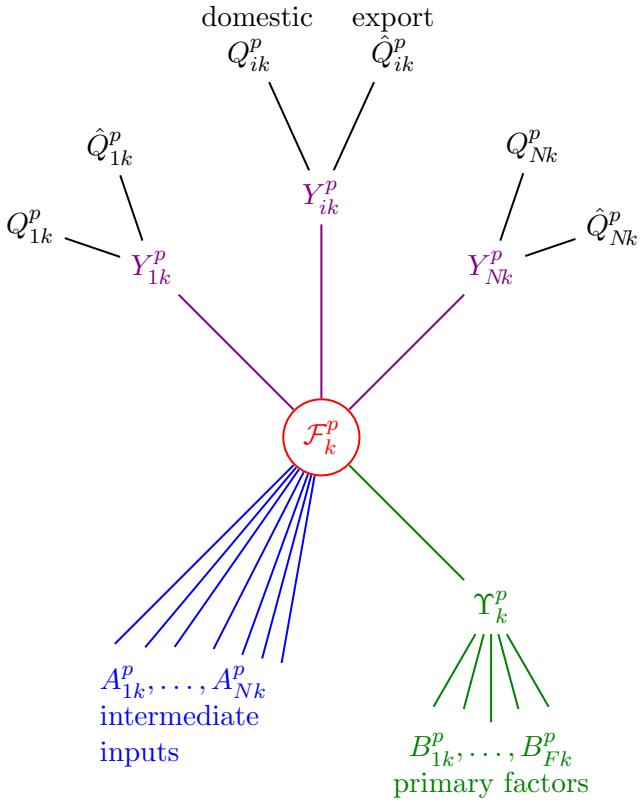
$$\texttt{TC}_k^p = \sum_{i=1}^{N} p_{ik}^p A_{ik}^p + \sum_{f=1}^{F} p_f^p B_{fk}^p \quad \text{(total cost)} \tag{2.19}$$

$$\texttt{TR}_k^p = \sum_{i=1}^{N} p_i^p Q_{ik}^p + \hat{p}_i^p \hat{Q}_{ik}^p \quad \text{(total revenue)} \tag{2.20}$$

$$\texttt{TC}_k^p = \texttt{TR}_k^p \quad \text{(zero profit)} \tag{2.21}$$

Equation (2.13) defines a constant-returns-to-scale multi-output production technology for each sector k in province p. On the output side, $\mathcal{F}_k^p(\cdot)$ is a CET transformation of 'unfinished' raw outputs $Y_{1k}^p,\ldots,Y_{Nk}^p$ which can be further processed into finished goods to be fed into the domestic-export transformation $Y_{ik}^p(\cdot)$ of equation (2.15). On the input side, $F_k^p(\cdot)$ is a two-level nested CES structure of intermediate inputs $A_{1k}^p,\ldots,A_{Nk}^p$ and a value-added function of primary factors $B_{1k}^p,\ldots,B_{Fk}^p$ (see figure 2.2).

Figure 2.2: Tree Diagram of Production Structure



After-tax producer prices are defined in equations (2.16, 2.17, 2.18) for primary factors, intermediate goods inputs, and final outputs. Taxes included rates from all levels of governments. Using these prices, equations (2.19, 2.20) calculate the total costs and revenues of production. Equation 2.21 sets the zero profit condition for optimal production.

2.5 Trade and Import Aggregation

The finished goods produced in equation (2.15) are appropriated for either domestic markets (Q^p_{ik}) or export markets ($\hat{Q}^p_{ik}$). There are going to be the goods purchased to create the Armington aggregates. The intermediate and final Armington composites are the same. In other words, when Ontario

consumers buy electrical products, they get the same share of Ontario to PEI to ROW electrical products as Ontario industry users do. This simplifies the model somewhat and we are unaware of any data to the contrary anyway.

Equations (2.22–2.23) describe the domestic-import Armington aggregation (see also figure 2.3). The nesting structure separates world imports M_i^{Wp} from domestic composites D_i^p which are in turns built from domestic goods and imports from other provinces. Hence, both world trade and inter-provincial trade can be incorporated into the model. It is assumed that there is no trade within provincial boundaries (i.e., $M_i^{pp} = 0$ in the same province p).

$$\mathcal{A}_i^p = \mathcal{A}_i^p(M_i^{Wp}, D_i^p) \tag{2.22}$$

$$D_i^p = D_i^p(Q_i^p, M_i^{1p}, \ldots, M_i^{Pp}) \tag{2.23}$$

Trade taxes are included in the price equation (2.24) for all regions r (i.e., provinces plus ROW). Negative values refer to trade subsidies such as export subsidies and import subsidies. Since there is no trade within provincial boundaries, trade taxes are not applicable in those situations.

$$\pi_i^{sd} = \begin{cases} \pi_i^s(1 + tx_i^{sd})(1 + tm_i^{sd}) & \forall\ d \neq s \quad \text{(from other regions)} \\ \\ \pi_i^s & \forall\ d = s \quad \text{(from own regions)} \end{cases} \tag{2.24}$$

2.6 Rest of World

Rest of world (W) has two types of activities:

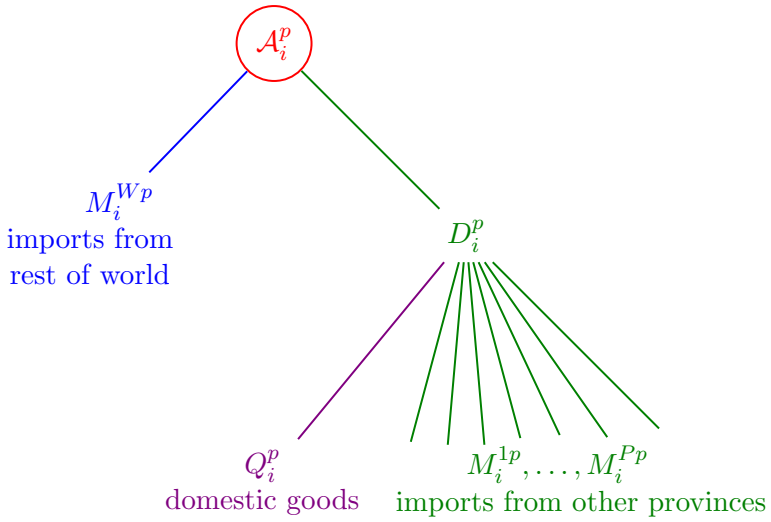
- Export activities (1 per commodity) take inputs of foreign exchange and produce ROW exports; in other words, Canada spends foreign exchange to acquire exports originated from ROW.
- Import activities (1 per commodity per province) take inputs of Canadian provincial exports and produce foreign exchange; in other words, Canada earns foreign exchange from exporting goods to ROW.

Equations (2.25-2.26) describe the 'technologies' embodying the small open economy assumption in these two activities.

$$\texttt{VWX}_i = \alpha_i \sum_{p=1}^{P} M_i^{Wp} \quad \text{(ROW export)} \tag{2.25}$$

$$\texttt{VWM}_i^p = \gamma_i^p\, M_i^{pW} \quad \text{(ROW import)} \tag{2.26}$$

Figure 2.3: Tree Diagram of Domestic-Import Aggregation



The proportional constant α_i is the price of imports of good i from the world (in foreign currency). It could also be though of as the quantity of foreign exchange required to 'produce' one unit of imports. If π_i^W is the import price in domestic currency, then equation (2.27) defines the relationship between these two prices:

$$\varepsilon = \frac{\pi_i^W}{\alpha_i} = \frac{\pi_i^{pW}}{\gamma_i^p} \tag{2.27}$$

Similarly, the proportional constant γ_i^p is the price of exports of good i from province p to the world (in foreign currency) and is linked to the export price π_i^{pW} in domestic currency by the foreign exchange rate ε as well.

$$\varepsilon \, \texttt{VWX}_i = \pi_i^W \sum_{p=1}^{P} M_i^{Wp} \tag{2.28}$$

$$\varepsilon \sum_{p=1}^{P} \texttt{VWM}_i^p = \sum_{p=1}^{P} \pi_i^{pW} M_i^{pW} \tag{2.29}$$

$\texttt{VWX}_i$ denotes the value in foreign currency of ROW exports to Canada while $\texttt{VWM}_i^p$ denotes the value in foreign currency of ROW imports from

province p. Equations (2.28-2.29) describe the zero profit conditions underlying the export-import activities of the world.

2.7 Market Clearing

Distinct markets exist for a number of composite commodities (those that are actually consumed by producers or consumers) as well as the produced goods from which they are derived. Factors of production are assumed to be mobile between sectors within a region, but inter-regionally immobile.

$$\sum_{k=1}^{K} A_{ik}^{p} + X_{i}^{p} + \psi_{i}^{p} + \sum_{l=1}^{L} \chi_{i}^{p\ell} + \sum_{l=1}^{L} \phi_{i}^{p\ell} = \mathcal{A}_{i}^{p} \tag{2.30}$$

$$Q_{i}^{p} = \sum_{i=1}^{K} Q_{ik}^{p} \tag{2.31}$$

$$\sum_{d=1}^{R} M_{i}^{pd} = \sum_{i=1}^{K} \hat{Q}_{ik}^{p} \tag{2.32}$$

$$\sum_{k=1}^{K} B_{fk}^{p} = E_{f}^{p} - \Lambda_{f}^{p} \tag{2.33}$$

$$\sum_{p=1}^{P} \left(\xi^{p} + \sum_{i=1}^{N} \texttt{VWM}_{i}^{p} \right) = \sum_{i=1}^{N} \texttt{VWX}_{i} \tag{2.34}$$

Equations (2.30-2.34) describe the market clearing conditions for all provinces (or regions). Each equation has the supply on its left hand side and the demand on its right hand side.

- Equation (2.30) requires that the total demand for Armington composites for intermediate input usages, consumer purchases (consumption plus investment), and government purchases (consumption plus investment by all levels) must equal its total supply.
- Equation (2.31) requires that for each good i in province p, the total demand for finished goods for domestic usages must equal its total supply over all sectors.
- Equation (2.32) requires that for each good i in province p, the total demand of finished goods for exporting to destination regions d must equal its total supply over all sectors. This condition covers both inter-provincial trade ($d = 1, \ldots, P$) and world trade ($d = W$). As mentioned on page 9, there is no trade inside the same region (i.e., $M_{i}^{dd} = 0$).

- Equation (2.33) requires that for each factor f in province p, the total demand of factors by all sectors must equal its factor endowment (net of leisure demand). As mentioned on page 5, there is no leisure demand for factors other than labor (i.e., $\Lambda_f^p = 0$ for $f \neq L$).
- Equation (2.34) requires that the total amount of foreign exchange needed for importing goods (including the consumer endowment portion ξ^p) must equal the total amount of foreign exchange earnings from exporting goods. In other words, the balance of payment must be in equilibrium.

2.8 Accounting Identities

This section provides additional details on various components of government revenues $\mathtt{GR}^{p\ell}$ needed for the right hand side of the government budget constraint (2.12). In general, each government level ℓ in province p receives tax revenues

$$\mathtt{GR}^{p\ell} \equiv \mathtt{Td}^{p\ell} + \mathtt{Te}^{p\ell} + \mathtt{Tf}^{p\ell} + \mathtt{Ti}^{p\ell} + \mathtt{Ty}^{p\ell} + \mathtt{Tm}^{p\ell} + \mathtt{Tx}^{p\ell} \tag{2.35}$$

from the following sources:

- Direct factor income taxes on consumer factor endowment incomes

$$\mathtt{Td}^{p\ell} = \sum_{f=1}^{F} td_f^{p\ell} w_f^p \left(E_f^p - \Lambda_f^p\right) \tag{2.36}$$

- Expenditure (sales) taxes on consumption expenditures (C) and investment expenditures (I). Taxes are levied on both private expenditures by consumers and public expenditures by governments.

$$\mathtt{Te}^{p\ell} = \mathtt{Te}_C^{p\ell} + \mathtt{Te}_I^{p\ell} \tag{2.37}$$

$$\mathtt{Te}_C^{p\ell} = \sum_{i=1}^{N} \pi_i^p \, te_{i_{\mathrm{C}}}^{p\ell} \left(X_i^p + \chi_i^{p\ell}\right) \tag{2.38}$$

$$\mathtt{Te}_I^{p\ell} = \sum_{i=1}^{N} \pi_i^p \, te_{i_{\mathrm{I}}}^{p\ell} \left(\psi_i^p + \phi_i^{p\ell}\right) \tag{2.39}$$

- Factor input taxes on producer factor usages in all sectors

$$\mathtt{Tf}^{p\ell} = \sum_{k=1}^{K} \sum_{f=1}^{F} tf_f^{p\ell} \, w_f^p \, B_{fk}^p \tag{2.40}$$

- Intermediate input taxes on producer intermediate usages in all sectors

$$\texttt{Ti}^{p\ell} = \sum_{k=1}^{K} \sum_{i=1}^{N} ti_{ik}^{p\ell}\, \pi_i^p\, A_{ik}^p \tag{2.41}$$

- Output taxes on producer outputs in all sectors

$$\texttt{Ty}^{p\ell} = \sum_{k=1}^{K} \sum_{i=1}^{N} ty_{ik}^{p\ell}\, \pi_i^p\, Q_{ik}^p \tag{2.42}$$

- Import tariffs on interprovincial and world trade (federal/provincial)

$$\texttt{Tm}^{p\ell} = \sum_{i=1}^{N} \sum_{s=1}^{R} (1 + tx_i^{sp\ell})\, tm_i^{sp\ell}\, \pi_i^p\, M_i^{sp} \tag{2.43}$$

- Export taxes on interprovincial and world trade (federal/provincial)

$$\texttt{Tx}^{p\ell} = \sum_{i=1}^{N} \sum_{d=1}^{R} tx_i^{pd\ell}\, \pi_i^p\, M_i^{pd} \tag{2.44}$$

Bibliography

[1] Markusen, James R. (2002). "Model M4_2S: Small Open Economy Model With A Benchmark Trade Imbalance."
http://www.mpsge.org/markusen/m4.htm#M4_2S

[2] Nguyen, Trien T. and Randall M. Wigle (2007). "BMRT 2001 Verion 3.0: User Guide." Technical Report (October 2007), University of Waterloo and Wilfrid Laurier University.

[3] Nguyen, Trien T., Krista Phillips, Karen Huff, Tracy Snoddon, and Randall M. Wigle (2007). "CREAP 2001 Verion 3.0: Data Documentation and Methods." Technical Report (October 2007), University of Waterloo and Wilfrid Laurier University.

Appendix A

Reference Tables

A.1 Subscripts

General notes on subscripts and superscripts are provided in section 2.1. Table A.1 lists the subscripts and their dimensions used in the model.

Table A.1: List of Subscripts

$f = 1, ..., F$	Subscript for factors of production; in particular, labor and leisure are referred to by f=L.
$p = 1, ..., P$	Subscript for provinces.
$r = 1, ..., R$	Subscript for regions which consist of provinces plus rest of world (denoted by r=W) and hence R=P+1; the double subscript sd refers to the source region s and the destination region d.
$i = 1, ..., N$	Subscript for goods/services produced
$j, k = 1, ..., K$	Number of sectors.
$\ell = 1, ..., L$	Subscript for levels of governments; the number of government levels L should not be mistaken with the index L for labor and leisure.

A.2 Quantities

Table A.2 lists the quantity variables used in the model.

Table A.2: List of Quantity Variables

U^p	Utility
Γ^p	Composite goods for consumption purchases by consumer p
Ψ^p	Composite goods for investment purchases by consumer p
X_i^p	Demand for good i in consumption purchases by consumer p
ψ_i^p	Demand for good i in investment purchases by consumer p
Λ_L^p	Demand for leisure by consumer p
E_f^p	Endowment of factor f by consumer p
ξ^p	Endowment of foreign exchange by consumer p
Z^p	Endowment of foreign exchange by consumer p (converted into domestic currency by the exchange rate ε)
T^p	Transfers received by consumer p
Y^p	Income by consumer p
$\chi^{p\ell}$	Composite goods by government ℓ in province p
$\chi_i^{p\ell}$	Demand for good i in consumption purchases by government ℓ in province p
$\phi_i^{p\ell}$	Demand for good i in investment purchases by government ℓ in province p
$\mathtt{GE}^{p\ell}$	Government expenditure by government ℓ in province p
$\mathtt{GR}^{p\ell}$	Government revenue by government ℓ in province p
Y_{ik}^p	Output of good i by sector k in province p
Q_i^p	Output of good i in province p (domestic)
Q_{ik}^p	Output of good i by sector k in province p (domestic)
$\hat{Q}_{ik}^p$	Output of good i by sector k in province p (export)
Υ_k^p	Value-added function of sector k in province p
A_{ik}^p	Intermediate input of good i by sector k in province p
B_{fk}^p	Factor input of factor f by sector k in province p
$\mathtt{TC}_k^p$	Total cost of sector k in province p
$\mathtt{TR}_k^p$	Total revenue of sector k in province p

List of Quantity Variables (cont.)

$\mathcal{A}_i^p$	Armington composite good i in province p
D_i^p	Domestic composite good i in province p
M_i^{sd}	Imports of good i from source region s to destination region d
$\texttt{VWX}_i$	Total input of foreign exchange into production of ROW exports of good i (i.e., foreign exchange to be spent on importing good i from ROW)
$\texttt{VWM}_i^p$	Receipts of foreign exchange from ROW imports of good i from source province p (i.e., foreign exchange earnings by province p from exporting good i to ROW)
α_i	proportional constant in the definition of $\texttt{VWX}_i$
γ_i^p	proportional constant in the definition of $\texttt{VWM}_i^p$

A.3 Prices

Table A.3 lists the price variables used in the model.

Table A.3: List of Prices

w_f^p	price of factor f in province p
π_i^p	price of good i in province p
π_f^p	price of factor f in province p less direct factor tax
$\pi_{i_{\mathrm{C}}}^p$	price of good i in province p plus expenditure tax on consumption purchases
$\pi_{i_{\mathrm{I}}}^p$	price of good i in province p plus expenditure tax on investment purchases
p_f^p	price of factor f in province p plus producer factor tax
p_i^p	price of good i in province p plus producer output tax
p_{ik}^p	price of good i in province p plus producer intermediate tax on sector k
p_i^s	seller's (producer) price of domestic good i in region s
$\hat{p}_i^s$	seller's (producer) price of export good i in region s
π_i^{sd}	landed price of good i from region s in region d
ε	price of foreign exchange

A.4 Taxes

Table A.4 lists the tax variables used in the model.

Table A.4: List of Taxes

td_f^p	direct tax on factor f in province p
te_{C}^p	expenditure tax on consumption purchases in province p
te_{I}^p	expenditure tax on investment purchases in province p
tf_f^p	factor input tax on factor f in province p
ty_i^p	output tax on good i in province p
ti_{ik}^p	intermediate input tax on good i by sector k in province p
tm_i^{sd}	import tax on good i of region s imported by region d
tx_i^{sd}	export tax on good i of region s exported to region d
$td_f^{p\ell}$	direct tax by government ℓ on factor f in province p
$te_{i_{\mathrm{C}}}^{p\ell}$	expenditure tax by government ℓ on consumption purchases of good i in province p
$te_{i_{\mathrm{I}}}^{p\ell}$	expenditure tax by government ℓ on investment purchases of good i in province p
$tf_f^{p\ell}$	factor input tax by government ℓ on factor f in province p
$ty_{ik}^{p\ell}$	output tax by government ℓ on good i by sector k in province p
$ti_{ik}^{p\ell}$	intermediate input tax by government ℓ on good i by sector k in province p
$tm_i^{sd\ell}$	import tax by government ℓ on good i of region s imported by region d
$tx_i^{sd\ell}$	export tax by government ℓ on good i of region s exported to region d
$\mathtt{Td}^{p\ell}$	direct tax revenues by government ℓ in province p
$\mathtt{Td}^{p\ell}$	expenditure tax revenues by government ℓ in province p
$\mathtt{Tf}^{p\ell}$	factor input tax revenues by government ℓ in province p
$\mathtt{Ty}^{p\ell}$	output input tax revenues by government ℓ in province p
$\mathtt{Ti}^{p\ell}$	intermediate input tax revenues by government ℓ in province p
$\mathtt{Tm}^{p\ell}$	import tax revenues by government ℓ in province p
$\mathtt{Tx}^{p\ell}$	export tax revenues by government ℓ in province p

A.5 Equations

Table A.5 lists the equations in each sector of chapter 2.

Table A.5: List of Equations

Section 2.2: Representative Agents	
Utility	2.1
Goods composites	2.2, 2.3
Consumer income	2.4
Budget constraint	2.5
Consumer prices	2.6, 2.7, 2.8
Foreign exchange endowment	2.9
Section 2.3: Governments	
Goods composites	2.10
Government expenditure	2.11
Government budget constraint	2.12
Section 2.4: Firms and Production	
Multi-output production	2.13
Value-added	2.14
Domestic-export transformation	2.15
Producer prices	2.16, 2.17, 2.18
Costs and revenues	2.19, 2.20
Zero profits	2.21
Section 2.5: Trade and Import Aggregation	
Armington aggregation	2.22
Domestic-import aggregation	2.23
Trade prices	2.24
Section 2.6: Rest of World	
ROW export-import technology	2.25, 2.26
Foreign exchange links	2.27
ROW zero profits	2.28, 2.29
Section 2.7: Market Clearing	
Market for Armington composites	2.30

List of Equations (cont.)

Market for domestic goods	2.31
Market for export goods	2.32
Market for factors	2.33
Market for foreign exchange	2.34
Section 2.8: Accounting Identities	
Government revenues	2.35
Revenues from direct factor income taxes	2.36
Revenues from expenditure taxes	2.37, 2.38, 2.39
Revenues from factor input taxes	2.40
Revenues from intermediate input taxes	2.41
Revenues from output taxes	2.42
Revenues from trade taxes	2.43, 2.44